

Problem 2.30

[Difficulty: 4]

2.30 Consider the flow field $\vec{V} = ax\hat{i} + b\hat{j}$, where $a = 1/4 \text{ s}^{-2}$ and $b = 1/3 \text{ m/s}$. Coordinates are measured in meters. For the particle that passes through the point $(x, y) = (1, 2)$ at the instant $t = 0$, plot the pathline during the time interval from $t = 0$ to 3 s. Compare this pathline with the streakline through the same point at the instant $t = 3$ s.

Given: Velocity field

Find: Plot of pathline for $t = 0$ to 3 s for particle that started at point (1,2) at $t = 0$; compare to streakline through same point at the instant $t = 3$

Solution:

Governing equations: For pathlines $u_p = \frac{dx}{dt}$ $v_p = \frac{dy}{dt}$

Following the discussion leading up to Eq. 2.10, we first find equations for the pathlines in form

$$x_p(t) = x(t, x_0, y_0, t_0) \quad \text{and} \quad y_p(t) = y(t, x_0, y_0, t_0)$$

$$x_{st}(t_0) = x(t, x_0, y_0, t_0) \quad \text{and} \quad y_{st}(t_0) = y(t, x_0, y_0, t_0)$$

which gives the streakline at t , where x_0, y_0 is the point at which dye is released (t_0 is varied from 0 to t)

Assumption: 2D flow

For pathlines $u_p = \frac{dx}{dt} = a \cdot x \cdot t$ $a = \frac{1}{4} \frac{1}{s^2}$ $b = \frac{1}{3} \frac{m}{s}$ $v_p = \frac{dy}{dt} = b$

So, separating variables $\frac{dx}{x} = a \cdot t \cdot dt$ $dy = b \cdot dt$

Integrating $\ln\left(\frac{x}{x_0}\right) = \frac{a}{2} \cdot (t^2 - t_0^2)$ $y - y_0 = b \cdot (t - t_0)$

$$x = x_0 \cdot e^{\frac{a}{2} \cdot (t^2 - t_0^2)} \quad y = y_0 + b \cdot (t - t_0)$$

The pathlines are $x_p(t) = x_0 \cdot e^{\frac{a}{2} \cdot (t^2 - t_0^2)}$ $y_p(t) = y_0 + b \cdot (t - t_0)$

where x_0, y_0 is the position of the particle at $t = t_0$. Re-interpreting the results as streaklines:

The pathlines are then $x_{st}(t_0) = x_0 \cdot e^{\frac{a}{2} \cdot (t^2 - t_0^2)}$ $y_{st}(t_0) = y_0 + b \cdot (t - t_0)$

where x_0, y_0 is the point at which dye is released (t_0 is varied from 0 to t)

Streakline and Pathline Plots

